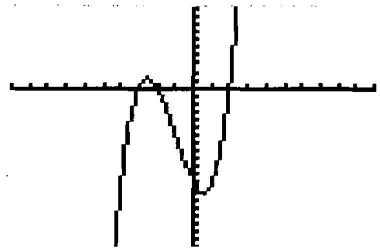
Answer Key - Part 1



1) $f(x) = x^3 + 3x^2 - 4x - 12$
Possible Zeros: ±12, ±6, ±4, ±3, ±2, ±1
f(x) = + + - - → one sign change means one positive zero
f(-x) = - + + - → two sign changes means two or zero negative zeros.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -4 & -12 \\ & \downarrow & 1 & 4 & 0 \\ \hline & 1 & 4 & 0 & -12 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & \downarrow & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$x = 2$

$x^2 + 5x + 6 = 0$
$(x+2)(x+3) = 0$
$x = -2$
$x = -3$

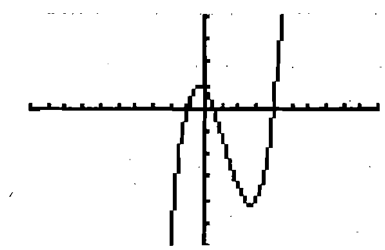
points on graph
(2,0)
(-2,0)
(-3,0)
(0,-12)
(1,-12)

Factored Form: $(x - 2)(x + 2)(x + 3)$

Zeros: $x = 2$, $x = -2$, $x = -3$

End Behavior: As x → $\underline{\infty}$, f(x) → $\underline{\infty}$. As x → $\underline{-\infty}$, f(x) → $\underline{-\infty}$.

2) $g(x) = 2x^3 - 7x^2 - 5x + 4$
Possible Zeros: $\pm 4, \pm 2, \pm 1, \pm \frac{1}{2}$



y-axis count by 5

f(x) = + - - + → two sign changes means two or zero positive zeros
f(-x) = - - + + → one sign change means one negative zero

$$\begin{array}{r|rrrr} 1 & 2 & -7 & -5 & 4 \\ & \downarrow & 2 & -5 & -10 \\ \hline & 2 & -5 & -10 & -6 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & -7 & -5 & 4 \\ & \downarrow & -2 & 9 & -4 \\ \hline & 2 & -9 & 4 & 0 \end{array}$$

$x = -1$

$2x^2 - 9x + 4 = 0$
$(x-4)(2x-1) = 0$
$x = 4$
$x = 1/2$

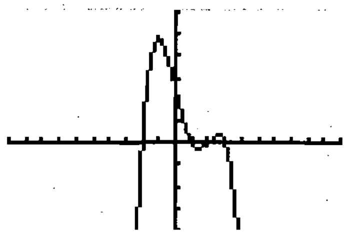
points on graph
(-1,0)
(4,0)
(1/2,0)
(0,4)
(1,-6)

Factored Form: $(x - 4)(x + 1)(2x - 1)$

Zeros: $x = 4$, $x = -1$, $x = \frac{1}{2}$

End Behavior: As x → $\underline{\infty}$, f(x) → $\underline{\infty}$. As x → $\underline{-\infty}$, f(x) → $\underline{-\infty}$.

Answer Key – Part 1



y-axis count by 5

points on graph

(1,0)
(2,0)
(-2,0)
(3,0)
(0,12)

3) $h(x) = -x^4 + 4x^3 + x^2 - 16x + 12$

Possible Zeros: ±12, ±6, ±4, ±3, ±2, ±1

f(x) = - + + - + → three sign change means three or one positive zero

f(-x) = - - + + + → one sign changes means one negative zero.

1	-1	4	1	-16	12
	↓	-1	3	4	-12
	-1	3	4	-12	0

2	-1	3	4	-12
	↓	-2	2	12
	-1	1	6	0

-1	-1	3	4	-12
	↓	1	-4	0
	-1	4	0	-12

$-x^2+x+6=0$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3$

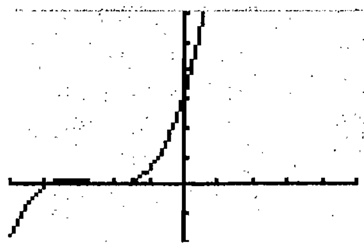
$x=-2$

Factored Form: -(x – 3)(x – 2)(x – 1)(x + 2)

Zeros: x = -2, x = 3, x = 2, x = 1

End Behavior: As x → ∞, f(x) → $-\infty$. As x → $-\infty$, f(x) → $-\infty$.

4) $j(x) = x^3 + 8x^2 + 20x + 16$



y-axis count by 5

points on graph

(1,45)
(-1,3)
(-2,0)
(-4,0)
(0,16)

Possible Zeros: ±16, ±8, ±4, ±2, ±1

f(x) = + + + + → zero sign changes means zero positive zeros

f(-x) = - + - + → two sign changes means two or zero negative zeros

1	1	8	20	16
	↓	1	9	29
	1	9	29	45

← oops, I shouldn't have tried +1 anyway because of Descartes' Rule of signs.

← upper bound

-1	1	8	20	16
	↓	-1	-7	-13
	1	7	13	3

-2	1	8	20	16
	↓	-2	-12	-16
	1	6	8	0

$x^2+6x+8=0$

$(x+2)(x+4)=0$

$x=-2$

$x=-4$

Factored Form: (x + 2)(x + 2)(x + 4)

Zeros: x = -2 (even multiplicity), x = -4

End Behavior: As x → ∞, f(x) → ∞. As x → $-\infty$, f(x) → $-\infty$.

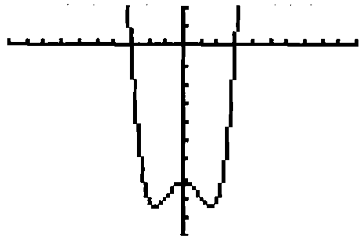
Answer Key – Part 1

5) $r(x) = x^4 - 5x^2 - 36$

Possible Zeros: ±36, ±18, ±12, ±9, ±6, ±4, ±3, ±2, ±1

$f(x) = + - -$ → one sign change means one positive zero

$f(-x) = + - -$ → one sign changes means one negative zero.



y-axis count by 5

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -5 & 0 & -36 \\ & \downarrow & 1 & 1 & -4 & -4 \\ \hline & 1 & 1 & -4 & -4 & -40 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 3 & 4 & 12 \\ & \downarrow & -1 & -2 & -2 \\ \hline & 1 & 2 & 2 & 10 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -5 & 0 & -36 \\ & \downarrow & 2 & 4 & -2 & -4 \\ \hline & 1 & 2 & -1 & -2 & -40 \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & 3 & 4 & 12 \\ & \downarrow & -2 & -2 & -4 \\ \hline & 1 & 1 & 2 & 8 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -5 & 0 & -36 \\ & \downarrow & 3 & 9 & 12 & 36 \\ \hline & 1 & 3 & 4 & 12 & 0 \end{array} \qquad \begin{array}{r|rrrr} -3 & 1 & 3 & 4 & 12 \\ & \downarrow & -3 & 0 & -12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

points

(1, -40)

(2, -40)

(3, 0)

(-3, 0)

(0, -36)

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Factored Form: $(x - 3)(x + 3)(x - 2i)(x + 2i)$

Zeros: $x = 3$, $x = -3$, $x = 2i$, $x = -2i$

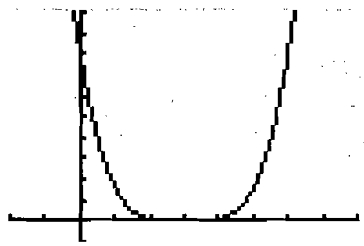
End Behavior: As $x \to \infty$, $f(x) \to \infty$. As $x \to -\infty$, $f(x) \to \infty$.

6) $p(x) = x^4 - 12x^3 + 54x^2 - 108x + 81$

Possible Zeros: ±81, ±27, ±9, ±3, ±1

$f(x) = + - + - +$ → four sign changes means 4, 2, or 0 positive zeros

$f(-x) = + + + + +$ → zero sign change means no negative zeros



y-axis count by 10

$$\begin{array}{r|rrrrr} 1 & 1 & -12 & 54 & -108 & 81 \\ & \downarrow & 1 & -11 & 43 & -65 \\ \hline & 1 & -11 & 43 & -65 & 16 \end{array} \qquad \begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & \downarrow & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -12 & 54 & -108 & 81 \\ & \downarrow & 2 & -20 & 68 & -80 \\ \hline & 1 & -10 & 34 & -40 & 1 \end{array}$$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3 \quad x = 3$

$$\begin{array}{r|rrrrr} 3 & 1 & -12 & 54 & -108 & 81 \\ & \downarrow & 3 & -27 & 81 & -81 \\ \hline & 1 & -9 & 27 & -27 & 0 \end{array}$$

points

(1, 16)

(2, 1)

(3, 0)

(0, 81)

$x = 3$ ✓✓✓✓

Factored Form: $(x - 3)(x - 3)(x - 3)(x - 3)$

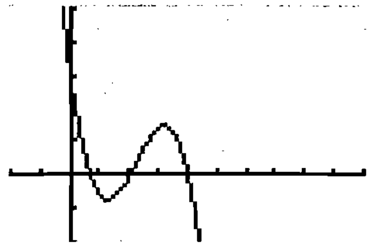
Zeros: $x = 3$ (even multiplicity)

End Behavior: As $x \to \infty$, $f(x) \to \infty$. As $x \to -\infty$, $f(x) \to \infty$.

Answer Key – Part 1

7) $n(x) = -3x^3 + 20x^2 - 36x + 16$

Possible Zeros: $\pm16, \pm8, \pm4, \pm2, \pm1, \pm\frac{16}{3}, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$



y-axis count by 5

$f(x)$ = - + - + → three sign changes means three or one positive zero

$f(-x)$ = + + + + → no sign changes means no negative zeros.

$$\begin{array}{r|rrrr} 1 & -3 & 20 & -36 & 16 \\ & \downarrow & -3 & 17 & -19 \\ \hline & -3 & 17 & -19 & -3 \end{array}$$

$$\begin{array}{r|rrrr} 2 & -3 & 20 & -36 & 16 \\ & \downarrow & -6 & 28 & -16 \\ \hline & -3 & 14 & -8 & 0 \end{array}$$

$-3x^2 + 14x - 8 = 0$

$\dfrac{-14 \pm \sqrt{196 - 4(-3)(-8)}}{-6}$

$\dfrac{-14 \pm \sqrt{196-96}}{-6} = \dfrac{-14 \pm \sqrt{100}}{-6} = \dfrac{-14 \pm 10}{-6}$

$\dfrac{-14+10}{-6} = \dfrac{2}{3}$ $\quad \dfrac{-14-10}{-6} = 4$

points

$(1,-3)$ $(2/3, 0)$

$(2,0)$ $(4,0)$

Factored Form: $-(x - 4)(x - 2)(3x - 2)$

Zeros: $x = 4$, $x = 2$, $x = 2/3$

End Behavior: As $x \to$ $\underline{\infty}$, $f(x) \to$ $\underline{-\infty}$. As $x \to$ $\underline{-\infty}$, $f(x) \to$ $\underline{\infty}$.

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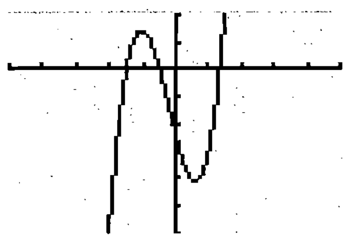
8) $m(x) = 12x^3 + 8x^2 - 23x - 12$

Possible Zeros: $\cancel{\pm12}, \cancel{\pm6}, \cancel{\pm4}, \cancel{\pm3}, \cancel{\pm2}, \cancel{\pm1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{2}{3}, \pm\frac{3}{4}, \pm\frac{4}{3}, \pm\frac{3}{2}$



y-axis count by 5

$f(x)$ = + + - - → one sign changes means one positive zero.

$f(-x)$ = - + + - → two sign change means two or one negative zeros

$$\begin{array}{r|rrrr} 1 & 12 & 8 & -23 & -12 \\ & \downarrow & 12 & 20 & -3 \\ \hline & 12 & 20 & -3 & -15 \end{array}$$

$\frac{12}{1} \cdot \frac{4}{3}$

$$\begin{array}{r|rrrr} 2 & 12 & 8 & -23 & -12 \\ & \downarrow & 24 & 64 & 82 \\ \hline & 12 & 32 & 41 & 70 \end{array}$$ ← upper bound

$\frac{4}{3} \cdot \frac{24}{1}$

$\frac{4}{3} \cdot \frac{9}{1}$

$$\begin{array}{r|rrrr} \frac{4}{3} & 12 & 8 & -23 & -12 \\ & \downarrow & 16 & 32 & 12 \\ \hline & 12 & 24 & 9 & 0 \end{array}$$

$12x^2 + 24x + 9 = 0$

$3(4x^2 + 8x + 3) = 0$

$3(2x+1)(2x+3) = 0$

$2x + 1 = 0 \quad x = -\frac{1}{2}$

$2x + 3 = 0 \quad x = -3/2$

points

$(1,-15)$

$(2,70)$

$(\frac{4}{3}, 0)$

$(-1/2, 0)$

$(-\frac{3}{2}, 0)$

Factored Form: $(2x + 1)(2x + 3)(3x - 4)$

Zeros: $x = -\frac{1}{2}$, $x = -3/2$, $x = 4/3$

End Behavior: As $x \to$ $\underline{\infty}$, $f(x) \to$ $\underline{\infty}$. As $x \to$ $\underline{-\infty}$, $f(x) \to$ $\underline{-\infty}$.

Answer Key - Part 2

1) $(x + 3)(x - 2)(2x - 1) = 2x^3 + x^2 - 13x + 6$

2) $(x - 4 - i)(x - 4 + i)(x - 2) = x^3 - 10x^2 + 33x - 34$

3) $(x - 4)(x - 3i)(x + 3i) = x^3 - 4x^2 + 9x - 36$

4) $(x + 5)(x + 5) = x^2 + 10x + 25$

5) $(x + 2)(x + 2)(x + 2) = x^3 + 6x^2 + 12x + 8$
The graph crosses at $x = -2$.

6) $(x + 4)(x + 4)(x - 2)(x - 2) = x^4 + 4x^3 - 12x^2 - 32x + 64$
The graph touches at both $x = 2$ and $x = 4$.

7) $(x - 1)(x + 3)(x + 3) = x^3 + 5x^2 + 3x - 9$